Are They Saying the Same Thing?

1.2

Using Patterns to Generate Algebraic Expressions

LEARNING GOALS

In this lesson, you will:

- Generate algebraic expressions using geometric patterns.
- Represent algebraic expressions in different forms.
- Determine whether expressions are equivalent.
- Identify patterns as linear, exponential, or quadratic using a visual model, a table of values, or a graph.

Are natural habits hard to break? The answer for most grocery stores would be, "Why in the world would we break these habits?" This is the reason why many grocery stores have followed a tried-and-true way for laying out their items in the aisles. Studies have shown that most Americans tend to prefer to shop in a counter-clockwise pattern; thus, most grocery stores have their produce at the front and to the right of the entrance which then leads (in a counter-clockwise manner) toward the bakery. And more cleverly, the bakery is toward the middle or the back of the store. From here, many stores lead you to the meat section and then the dairy section. So, why are the bakery, meat, and dairy sections toward the back of the store? Once again, grocery stores embrace people's natural tendencies. For most families, the most needed items are meats, breads, and milk. So, when these items are toward the back of the store, it provides more chances for customers to make "impulse" purchases along the way—buying things that weren't on the original grocery list!

While scientists don't know what causes this impulse (moving in a counter-clockwise manner, or buying items that aren't necessarily needed), it is extremely strong.

This impulse to move in a counter-clockwise direction can be thought of as a pattern similar to animal migrations. Is this the only way to get to where you are going? Of course not, but for some reason, it seems to be a more comfortable path. When problem solving in mathematics there are often many ways for you to approach a problem, but usually you choose a familiar method. Do you usually find one way to do something and then stick with it, or do you look for different methods?

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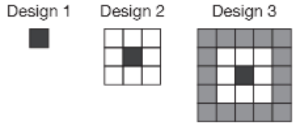
PROBLEM 1 Floors by Terrance

Terrance's flooring business from the problem, *There's More Than One Way to Tile a Floor*, was booming! He decides to hire several employees to help lay out his tile designs. It will be necessary for Terrance to describe his tile designs in a clear manner so that all of the employees can create them correctly. Recall that Terrance's square floor design uses alternating black, white, and gray tiles.



1. Describe the pattern in terms of the number of new tiles that must be added to each new square floor design.

2. Write an expression to represent the number of new tiles that must be added to an n by n square floor design. Let n represent the number of tiles along each edge of the square.

3. Describe which values for n make sense in this problem situation?

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4. Ramone determined an expression to represent this pattern. His expression and explanation are shown.

Ramone

Design	1	2	3
New Tiles	0	8	16

The expression $8(n - 1)$ represents Terrance's square floor pattern. I noticed that the number of new tiles is increasing by 8 in each new design.

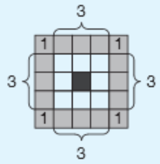
Explain why Ramone's expression is incorrect.

5. Describe the pattern as new tiles are added as linear, quadratic, exponential, or none of these. Explain your reasoning.

6. Terrance asks his employees to determine the number of new tiles added to Design 2 to create Design 3. Each employee describes a unique method to determine the number of additional tiles needed to create Design 3. Represent each of his employee's explanations with an algebraic expression that describes how many new tiles must be added to an $n \times n$ square to build the next design.

Wilma

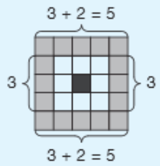
I must add 3 tiles to each of the four sides of the white square, which is $4 \cdot 3$ tiles. Then I must add 1 tile at each corner. So the number of additional tiles added to a Design 2 square floor design is $4 \cdot 3 + 4$.



Expression: ______________________________

Howard

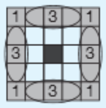
I must add 5 tiles to two of the sides and 3 tiles to the other two sides. The number of additional tiles added to Design 2 square floor design is $2(3 + 2) + 2 \cdot 3$.



Expression: ______________________________

Tyler

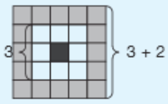
I need to add 3 tiles four times and then add the four corner tiles. The number of additional tiles added to Design 2 square floor design is 3 + 3 + 3 + 3 + 4.



Expression: ______________________________

Tamara

The way I look at it, I really have two squares. The original square for Design 2 has 3 · 3 tiles. The newly formed Design 3 square has 5 · 5 tiles. So, the number of additional tiles added to the Design 2 square floor design is 5 · 5 − 3 · 3.



Expression: ______________________________

7. Which expression do you think Terrance should use? Explain your reasoning.

Does the expression you determined match one of the expressions Terrance's employees determined?

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8. Michael and Louise analyze the expressions they wrote for each student. They both determined that the expression to represent Tamara's method is $(n + 2)^2 - n^2$. Michael claims that this expression is quadratic because of the n^2 term. Louise disagrees and says the expression is linear because the pattern is linear.
Who is correct? Explain your reasoning.

9. Use each expression you determined in Question 6 to calculate the number of tiles that must be added to squares with side lengths of 135 tiles to create the next design.

Wilma's expression:

Tyler's expression:

Howard's expression:

Tamara's expression:

10. Wilma tells Terrance that since all of the expressions resulted in the same solution, any of the expressions can be used to determine the number of additional tiles needed to make more $n \times n$ designs. Terrance thinks that the employees need to use more values in the expressions than just one to make this conclusion.
Who is correct? Explain your reasoning.

Recall that two or more algebraic expressions are equivalent if they produce the same output for all input values. You can verify that two expressions are equivalent by using properties to rewrite the two expressions as the same expression.

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11. Show that Wilma, Howard, Tyler, and Tamara's expressions are equivalent. Justify your reasoning.

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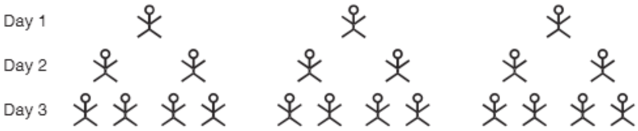
PROBLEM 2 The Cat's Out of the Bag!

Let's revisit the problem, *Can You Keep a Secret?* about the homecoming king election. The visual model shown represents the number of new seniors who learn about the election result each day that passes.

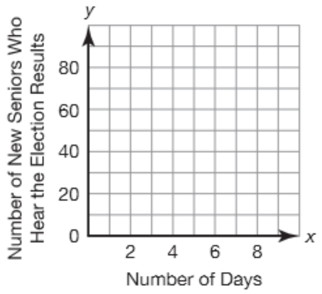


1. Analyze the pattern.

   a. Complete the table to summarize the number of new seniors who learn about the election result each day. Then write an expression to represent the number of new seniors who learn about the election result on the nth day. Finally, describe how each part of your expression relates to the visual model.

Number of Days That Pass	Number of Seniors Who Hear the Results That Day
1	
2	
3	
4	
5	
6	
n	

b. Create a graph of the data from your table on the coordinate plane shown. Then draw a smooth curve to model the relationship between the number of days that pass and the number of seniors who hear the senior election results.



2. Do all the points on the smooth curve make sense in terms of this problem situation? Why or why not?

3. Describe this pattern as linear, exponential, quadratic, or none of these. Then write the corresponding equation. How does each representation support your answer?

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4. Describe the key characteristics of your graph. Explain each characteristic algebraically and in terms of this problem situation.

5. After how many days will 500 new seniors learn about the election results?

6. Determine the number of seniors who hear the election results on the twelfth day. Does your answer make sense in the context of this problem? Explain your reasoning.

PROBLEM 3 Several Spreading Sequences of Squares

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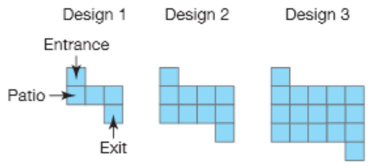
Let's revisit the problem, *How Large Is Your Yard?* about backyard patio designs. The model shown represents the first three designs Maureen and Matthew could use. Each square represents 1 square foot.



1. Determine the number of squares in the next two patio designs of the pattern.

2. Write an expression to determine the total number of squares in patio Design n. Describe how each part of your expression relates to the visual model.

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3. Maureen and Matthew each write different expressions to represent the patio designs.

a.

Maureen

$(n + 2)^2 - 2(n + 1)$

Describe how each term in Maureen's expression represents the visual model.

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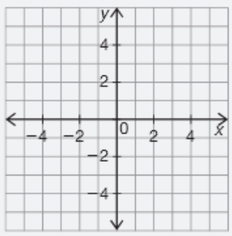
b.

Matthew

$n^2 + 2n + 2$

Use a graphing calculator to graph each expression. Is Matthew's expression correct? Explain your reasoning in terms of the graph.

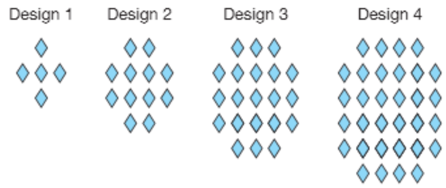


4. Identify the parts of the graph that represent this problem situation.

5. In order to accommodate outdoor furniture, a grill, and a shed, the patio must have an area of at least 125 square feet (not including the walkways). What is the smallest design Matthew can build and still have enough space for these items?

Talk the Talk

1. Analyze the pattern shown.



a. Identify two expressions that represent the total number of diamonds used to construct Design *n*.

b. Describe how your expressions relate to the visual model.

c. Algebraically prove your expressions are equivalent.

d. Graphically show that your expressions are equivalent.

2. Describe the ways you can prove any two expressions are equivalent.

Be prepared to share your solutions and methods.